CS135 Section 9.6

page 630/3 (c, d) is (S, R) is a poset?

S is the set of all people in the world.

c) $(a, b) \in R$ if $a, b \in S$ and $a = b$ or a is an ancestor of b.

- reflexive, because $(a, a) \in R$ since $a = a$
- antisymmetric, because if $(a, b) \in R$ and $(b, a) \in R$

  we get that ($a = b$ or a is ancestor of b) and

  contradict to each other

  ($b = a$ or b is ancestor of a)

  so only case $a = b$ is possible
- transitive, because if (a, b) and $(b, c) \in R$ then we get:

  ($a = b$ or a is ancestor of b) and ($b = c$ or b is ancestor of c)

  - so $a = b$ and b is ancestor of c can happen, so a is ancestor of c, hence $(a, c) \in R$
  - or $a = b$ and $b = c$, so $a = c$ and $(a, c) \in R$
  - or a is ancestor of b, and $b = c$, so a is ancestor of c, $(a, c) \in R$
  - or a is ancestor of b, b is ancestor of c, hence a is ancestor of c, i.e. $(a, c) \in R$.

Therefore, the given pair (S, R) is a poset.

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page 630/3 (d)

d) $(a, b) \in R$ if a and b have a common friend.

- it is not a poset, because it is not transitive.

If a and b have a common friend, it doesn't mean that if b and c have a common friend, too, that common friends are the same.

(it is not asymmetric either)

page 630/5 (a, b)

a) $(\mathbb{Z}, =)$ is a poset, because relation "=" is

- reflexive, i.e. $a = a$
- asymmetric, i.e. if $a = b$ and $b = a$, then they are the same
- transitive, i.e. if $a = b$ and $b = c$, then $a = c$.

b) $(\mathbb{Z}, \neq)$ is not a poset, because it is not transitive

$\underset{a}{2} \neq \underset{b}{3}$, $\underset{b}{3} \neq \underset{c}{2}$ but $\underset{a}{2} = \underset{c}{2}$

(it is not asymmetric either)